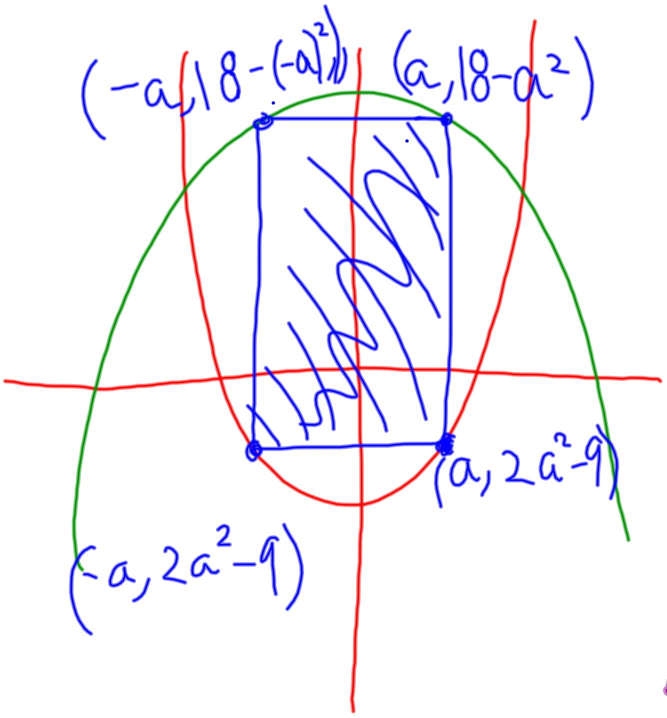
$f(x) = 18 - x^2$ $g(x) = 2x^2 - 9$

Find the inscribed rectangle of largest area inside the region.



bH

$(2a)(27-3a^2)$

$A = 54x - 6x^3$

$0 = 54 - 18x^2 = A'$

$x = \sqrt{3}$

1971 AB6 start: $(5,0)$ at $t=0$

$v(t)\ [t>0] = \frac{t}{1+t^2}$

a) max v & justify

b) position at $t=6$

c) limiting value of $v(t)$

d) does the particle ever pass $(500,0)$?

Jan 30-8:35 AM

a) $v(t)=\frac{t}{1+t^2}$ $\frac{1}{1+(1)^2}=\frac{1}{2}$

$v'(t)=\frac{1-t^2}{(1+t^2)^2}$

$v'(0)=\frac{1-t^2}{(1+t^2)^2}=0 \Rightarrow 1-t^2=0$, $t^2=1$, $t=\pm 1$ $t=1$

t can't be negative

Max Velocity $\frac{1}{2}$ at $t=1$

b. $v(t)=\frac{t}{1+t^2}$ $\int \frac{t}{1+t^2}\,dt \frac{du}{2t}$ $1+t^2=u$

$\frac{du}{dt}=2t$

$dt=\frac{du}{2t}$

$\frac{1}{2}\int \frac{t}{u}\frac{du}{2t}=\frac{1}{2}\int\frac{1}{u}du$

and you know $P(0)=5$

$\frac{1}{2}\ln u +C= P(t)$

so $\frac{1}{2}\ln(1+0^2)+C=5$ so $C=5$

$\frac{1}{2}\ln(1+t^2)+5=P(t)$ $P(t)=$ Position

$\frac{1}{2}\ln(1+36)=P(6)$

$\frac{1}{2}\ln 37+5=P(6)$

b) alternate

final distance (position) = starting distance + displacement [total chg in distance]

$P(6) = P(0) +$ add up all the instantaneous chgs in position

$P(6)=P(0)+\int_0^6 v(t)\,dt$

c) $\lim_{t\to\infty} v(t)=\lim_{t\to\infty}\frac{t}{1+t^2}=\lim_{t\to\infty}\frac{t(1)}{t(\frac{1}{t}+t)}=\lim_{t\to\infty}\frac{1}{\frac{1}{t}+t}=\frac{1}{\infty}=0$

or L'Hospital's Rule

d) does particle ever pass (500,0)?

But: $P(t)=5+\frac{1}{2}\ln(1+t^2)$

so $500=5+\frac{1}{2}\ln(1+t^2)$ does this have a solution?

$495=\frac{1}{2}\ln(1+t^2)$

$990=\ln(1+t^2)$ $e^{990}=1+t^2$

so $t=\pm\sqrt{e^{990}-1}$

Jan 31-7:40 AM